Performance Based Learning and Assessment Task

Alpaca Lunch

I. ASSESSMENT TASK OVERVIEW & PURPOSE:
Students will make and test conjectures regarding possible relationships between the lateral surface area and the volume of solids. Actual solids will be constructed to test conjectures and results verified using algebraic formulas.

II. UNIT AUTHOR:
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III. COURSE:
Geometry

IV. CONTENT STRAND:
Three Dimensional Figures

V. OBJECTIVES:
The learner will be able to use algebraic or geometric reasoning to maximize the volume of a three-dimensional figure given constraints on the lateral surface area of the figure.

VI. REFERENCE/RESOURCE MATERIALS:
Tape
Scissors
Paper plates
Classroom set of calculators
Formula Sheet for Geometry
8X10 sheets of paper
Heavy card stock or large paper plates
Popcorn, cereal, dried beans, foam packing material, rice, etc.
Handouts

VII. PRIMARY ASSESSMENT STRATEGIES:
The task includes an assessment component that the student may use as a checklist for self-assessment and the teacher may use as a grading rubric. The assessment component is comprised of the included rubric that provides clear details on grading criteria for student work.

VIII. EVALUATION CRITERIA:
- Each student will be evaluated using the included rubric.
- Benchmark of Exemplary Work will guide teachers for intended baseline of students' work

IX. INSTRUCTIONAL TIME:
90 minutes over 2 days

Alpaca Lunch

Strand
3-Dimensional Figures

Mathematical Objective(s)
- Students will use their knowledge of geometric shapes and formulas to maximize volume
- Students will use prior geometric knowledge to make conjectures before completing the activity prompt

Related SOL

G.13 (The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.)

G.14 (The student will use similar geometric objects in two- or three-dimensions to
a) compare ratios between side lengths, perimeters, areas, and volumes;
b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
d) solve real-world problems about similar geometric objects.)

NCTM Standards
- Analyze properties and determine attributes of two- and three-dimensional objects
- Apply and adapt a variety of appropriate strategies to solve problems
- Communicate mathematical thinking coherently and clearly to peers, teachers, and others
- Establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others

Materials/Resources
Tape
Scissors
Paper plates
Classroom set of calculators
Formula Sheet for Geometry
8X10 sheets of paper
Heavy card stock or large paper plates
Popcorn, cereal, dried beans, foam packing material, rice – some material to fill the solids
Handouts

Assumption of Prior Knowledge
- The student must know how to use the geometric formulas related to lateral area, area of base and volume, including finding the necessary lengths to calculate these formulas
- The student must be able to use mathematical concepts to defend or discredit his/her conjecture

Introduction: Setting Up the Mathematical Task

In this activity, you will investigate the relationship between the lateral surface area of a container and the amount of material the container will hold. On the first day, you will look at how lateral surface area can remain the same as students create containers with different heights and different bases Each container will be created using the same sized paper to form its sides. On the second day, you will use your knowledge to maximize the volume of the food container in order to provide customers with the best deal possible. You will establish a conjecture, use math to either support or discredit your conjecture and then thoroughly explain your design, math and reasoning.

Student Exploration

Individual Work (estimated time: 2 90-minute class periods)

- Students will work individually on the activity with assistance from the teacher when necessary

Student/Teacher Actions:

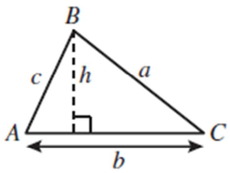
- Students should take into consideration the supplies available to complete this project
- Students should thoroughly examine the activity prompt and the provided rubrics to ensure they understand the expectations of the activity
- The teacher should circulate and ensure students understand what their goal is for the activity and that reasonable conjectures have been established.
- The teacher can provide help to students that are struggling by making suggestions on the shape of container students may want to test, various dimension sets to try testing and/or providing help with formula calculations.

Monitoring Student Responses

- Students should be supportive and respectful of all students during the activity. Providing help to struggling students should be encouraged, but should not simply be for the supplying of answers.
- If students need a modification to the activity, providing them with the various shapes and dimensions is an option.
- Students will be encouraged and expected to use mathematical vocabulary during the activity and in their reasoning.

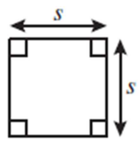
Geometry Formula Sheet
2009 *Mathematics Standards of Learning*

Geometric Formulas

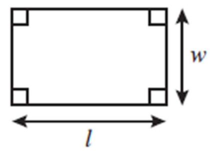


$$A = \frac{1}{2}bh$$

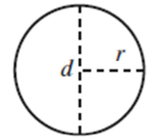
$$A = \frac{1}{2}ab \sin C$$



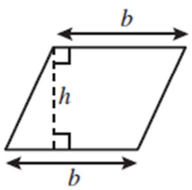
$$p = 4s$$
$$A = s^2$$



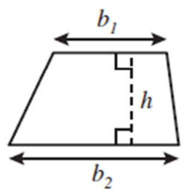
$$p = 2l + 2w$$
$$A = lw$$



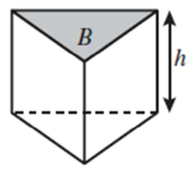
$$C = 2\pi r$$
$$C = \pi d$$
$$A = \pi r^2$$



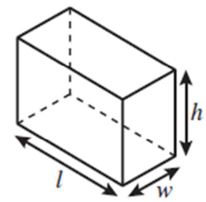
$$A = bh$$



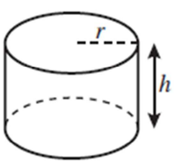
$$A = \frac{1}{2}h(b_1 + b_2)$$



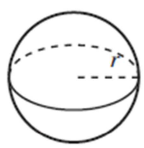
$$V = Bh$$
$$L.A. = hp$$
$$S.A. = hp + 2B$$



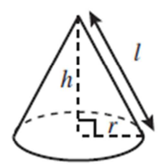
$$V = lwh$$
$$S.A. = 2lw + 2lh + 2wh$$



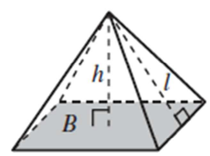
$$V = \pi r^2 h$$
$$L.A. = 2\pi rh$$
$$S.A. = 2\pi r^2 + 2\pi rh$$



$$V = \frac{4}{3}\pi r^3$$
$$S.A. = 4\pi r^2$$



$$V = \frac{1}{3}\pi r^2 h$$
$$L.A. = \pi rl$$
$$S.A. = \pi r^2 + \pi rl$$



$$V = \frac{1}{3}Bh$$

$$L.A. = \frac{1}{2}lp$$

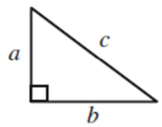
$$S.A. = \frac{1}{2}lp + B$$

Abbreviations

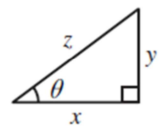
Area	A
Area of Base	B
Circumference	C
Lateral Area	$L.A.$
Perimeter	p
Surface Area	$S.A.$
Volume	V

Geometry Formula Sheet
2009 *Mathematics Standards of Learning*

Geometric Formulas



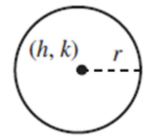
$a^2 + b^2 = c^2$



$\text{Sin } \theta = \frac{y}{z}$

$\text{Cos } \theta = \frac{x}{z}$

$\text{Tan } \theta = \frac{y}{x}$



$(x - h)^2 + (y - k)^2 = r^2$

Pi

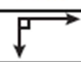
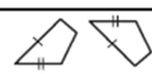
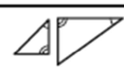
$\pi \approx 3.14$

$\pi \approx \frac{22}{7}$

Quadratic Formula:

$$x = \frac{^-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } ax^2 + bx + c = 0 \text{ and } a \neq 0$$

Geometric Symbols

Example	Meaning
$m\angle A$	measure of angle A
AB	length of line segment AB
$\overrightarrow{AB}$	ray AB
	right angle
$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	Line AB is parallel to line CD.
$\overline{AB} \perp \overline{CD}$	Line segment AB is perpendicular to line segment CD.
$\angle A \cong \angle B$	Angle A is congruent to angle B.
$\triangle ABC \sim \triangle DEF$	Triangle ABC is similar to triangle DEF.
	Similarly marked segments are congruent.
	Similarly marked angles are congruent.

*Optional
Pre-Activity Worksheet

Vocabulary Review

Area

Surface Area

Lateral Area

Volume

Exploration: The Petting Zoo

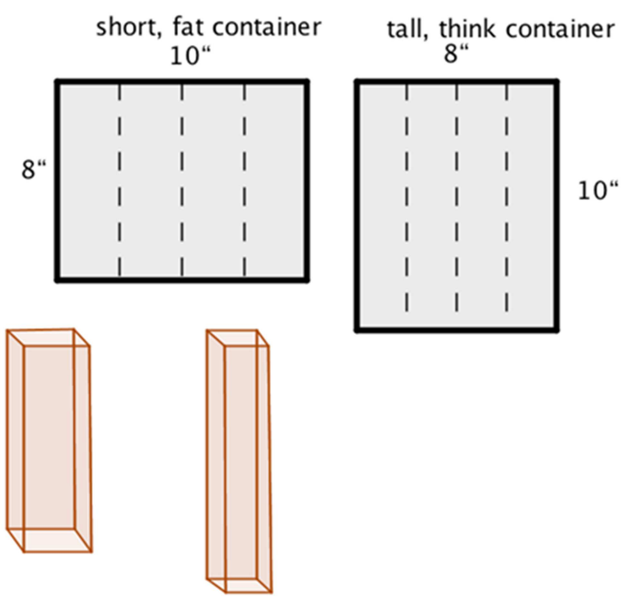
Congratulations! You have a part-time job at a local petting zoo. Because the zoo is free for children, it wants to earn some money selling grain for visitors to feed the alpaca and llamas.
A friend of the zoo has donated special paper to make alpaca food containers. You have many sheets of this paper, cut to 8” x 10”. These sheets will make the sides of your containers, and you will use heavy card stock to create the bottom.

One of your helpers used the paper to make tall, skinny containers.
Another helper began making short, fat containers.

A set of twin boys came to the zoo with their mother. “I want a tall, skinny container,” said one twin. “No, buy the short, fat one, “ said the other. Their mother said it doesn’t matter which one they buy, they both hold the same amount of grain.

Who is right? Make a conjecture now, and support your thinking.

Test your conjecture. Use your paper to create one of each type of container, as shown below.



Use the material provided by your teacher to fill one container. Will this amount fit exactly into the other container? Pour into the other to test our conjecture.

Did the experiment support your conjecture? Do you have the same opinion, or did you change your mind? Explain.

Let's use math to support what your opinion.

Analyze the short, fat container:

Length	Width	Height

Lateral area	Area of Base	Volume

Analyze the tall, thin container:

Length	Width	Height

Lateral area	Area of Base	Volume

Does the math support your original conjecture? Explain?

What is your final conclusion? Explain in detail and support your answer.

Activity Prompt
Alpaca Lunch

Good news! A generous donor has given the petting zoo free Alpaca Chow. Since the food is free, you have decided to give the zoo visitors the best deal you can. You want to make one container hold the most food it can without changing your materials.

You will have as much 8' X 10" paper as you want. You will use card stock for the bottom of your container. You can only use one sheet of paper to make the sides of each container, but you may make the base as large as you need to.

You may make your container any shape you wish – a rectangular prism, cylinder, or triangular prism – any shape that can be used to hold Alpaca Chow.

Make a conjecture: What shape do you think will hold the most food? Explain your thinking in detail.

Test your conjecture: Use your materials to test your conjecture. You may perform as many experiments as you wish.

Revise your conjecture: After experimenting, does your conjecture still hold? Do you wish to change your conjecture? Explain.

Support your reasoning with math: Use your formula sheet to check your revised conjecture. DO you need to revise your conjecture?

Final conclusion: What is your final conclusion? How can you use 80 square inches of lateral surface area and create the largest possible container? Address the following in your answer: What is the largest container that you tested or analyzed? Do you believe that it is the largest possible one? Support your answer with experiment results and math.

Assessment List – Rubric

	1	2	3	4	
Original conjecture	Conjecture was listed in vague terms and not supported.	Conjecture is vague but supported, or conjecture is clear and precise but unsupported.	Conjecture is clear and precise but explanation is not clear.	Original conjecture was described in detail and logically supported	/4
Test your conjecture	There is no evidence that conjecture was tested	There is evidence that one or two other options were explored	Options were explored, but counterexample to conjecture was not tested	Evidence exists that the conjecture was thoroughly tested	/4
Revise your conjecture	Incorrect conjecture was not revised with little or no explanation	Incorrect conjecture was not revised, explanation given is weak or incomplete	Correct conjecture is not sufficiently supported or is revised, incorrect conjecture is revised without support	Correct conjecture is supported, or incorrect conjecture is appropriately revised	/4
Support your reasoning	No math is shown, or math shown is incorrect	Only one container is analyzed mathematically	Math is shown, but does not support answer	Math is correct, and supports correct conjecture	/4
Final Conclusion	Final conclusion is not supported	Final conclusion is not the best solution, but is supported by either mathematics or experimentation	Final conclusion is not the best solution, but is supported by mathematics and experimentation	Final conclusion is correct and fully supported	/4
				Score	/20

Benchmark
Example of Exemplary Work

Good news! A generous donor has given the petting zoo free Alpaca Chow. Since the food is free, you have decided to give the zoo visitors the best deal you can. You want to make one container hold the most food it can hold without changing your materials.

You will have as much 8’ X 10” paper as you want. You will still use card stock for the bottom of your container. You can only use one sheet of paper to make the sides of each container, but you may make the base as large as you need to.

You may make your container any shape you wish – a rectangular prism, cylinder, triangular prism – any shape that can be used to hold Alpaca Chow.

Make a conjecture: What shape do you think will hold the most food? Explain your thinking in detail.

I think that a short, fat rectangular prism with a square base will hold the most food. When we did the activity yesterday that was the one that held the most.
The prism would be 8” tall, 2.5” wide and 2.5” deep.

Test your conjecture: Use your materials to test your conjecture. You may perform as many experiments as you wish.

I tried making cylinders and prisms that did not have squares on the bases. The cylinders that are 8” tall hold almost the same as the prisms that are 8” tall; it is hard to tell which one is bigger. It seems like the wider the container is, the more it will hold.

Revise your conjecture: After experimenting, does your conjecture still hold? Do you wish to change your conjecture? Explain.

I think that a short, far cylinder will hold more than the short fat rectangular prism. If we cut the paper in half, we can make a container that is very short and very wide that will hold even more.

Support your reasoning with math: Use your formula sheet to check your revised conjecture. Do you need to revise your conjecture?

8" tall cylinder: V = $\pi r^2 h$ Circumference = 10" = $2\pi r$ r= $\frac{5}{\pi} \approx 1.59$ V = $\pi * 1.59^2 * 8 \approx 63.5$ in^3	**8" tall prism:** V = l*w*h = 8*2.5*2.5 = 50 in^3
4" tall cylinder: C= 20" = $2\pi r$ r= $\frac{10}{\pi} \approx 3.18$ V= $\pi * 3.18^2 * 4 \approx 127.01$ in^3	**4"tall prism:** V = 4*5*5= 80 in^3

Final conclusion: What is your final conclusion? How can you use 80 square inches of lateral surface area and create the largest possible container? Address the following in your answer: What is the largest container that you tested or analyzed? Do you believe that it is the largest possible one? Support your answer with experiment results and math.

The largest container I could make was a cylinder that was 4" tall. That was not what I originally thought. I did not think that cylinders would hold so much more than prisms. I think it is because they do not have corners, they have larger bases. When I used the formulas, I could see that when I squared the radius of the base circle, that this made a big change in the volume of the container.